THE UNITED REPUBLIC OF TANZANIA

MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2023

BASIC APPLIED MATHEMATICS

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(ACSEE) 2023

141 BASIC APPLIED MATHEMATICS

Published by:

The National Examinations Council of Tanzania,

P.O. Box 2624,

Dar es Salaam, Tanzania.

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FOREWORD

This report is on the Candidates' Item Response Analysis (CIRA) for the 141 Basic Applied Mathematics paper of the Advanced Certificate of Secondary Education Examination (ACSEE) 2023. Hence, it serves as a feedback to education stakeholders on the factors that affected performance of the candidates.

The candidates performed well in *Statistics*, *Calculating Devices* and *Probability*. The good performance in these topics were attributed to the candidates' competencies in presenting data using histogram, calculating the median and percentiles and using a non-programmable scientific calculator to evaluate mathematical expressions, integral of a function and the roots of the polynomial function. The candidates also demonstrated competency in applying 'combinations' in daily life and determining the probability of combined events. The overall performance of the candidates in *Algebra, Functions* and *Matrices and Linear Programming* was average while the performance in *Exponential and Logarithmic Functions*, *Trigonometry*, *Integrations* and *Differentiation* was weak. The weak performance was attributed to the failure of the candidates to draw graphs of rational functions, interpret graphs of exponential functions, apply trigonometric identities and solve trigonometric equations. The candidates also were not conversant with the properties of definite integrals and the technique of evaluating the integrals of polynomials expressed in the form $(ax+b)^n$. Moreover, most candidates failed to solve the real-life problems related to variations and differentiation.

The National Examinations Council of Tanzania expects that education stakeholders (especially mathematicians) will consider this report as one of the reviews in planning effective strategies for improving the performance of the candidates in Mathematics.

Finally, the Council would like to thank all who participated in the preparation of this report.

Dr. Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The 141 Basic Applied Mathematics paper of ACSEE 2023 comprised ten (10) compulsory questions, each carrying ten (10) marks. The paper was set based on the Basic Applied Mathematics Syllabus for Advanced Secondary Education of 2010 and the Basic Applied Mathematics examination format of July 2019.

In the year 2023, a total of 38,360 candidates sat for the paper, of which 68.53 per cent passed. In 2022, about 58.99 per cent of the candidates passed. Therefore, the percentage of candidates who passed in 2023 increased by 9.87 per cent. Figure 1 shows the percentage of the candidates who got a particular grade in 2022 and 2023.

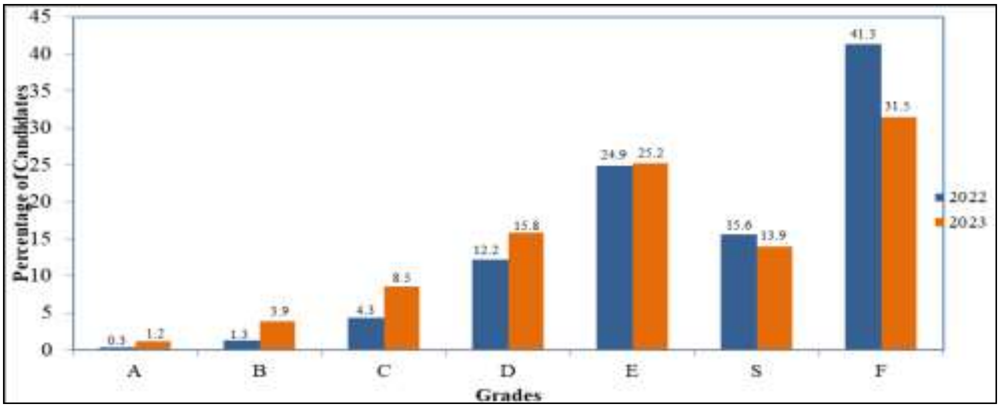


Figure 1: *Performance of the candidates in 2022 and 2023*

The percentage in grades A, B, C, D and E has increased in 2023 while that in grades S and F has decreased. Furthermore, the greater change has been observed in grade F whereby the percentage of candidates who got F has decreased by 9.8 per cent. This indicates that the percentage of the candidates with low marks has decreased while that of high marks has increased. Thus, there is improvement in candidates' performance in 2023 compared to the performance in 2022.

Section 2 of the report describes the performance of candidates in each question as well as the candidates' responses. In Section 3, the report shows the performance of the candidates in each topic, while Section 4 presents the conclusion and recommendations. Finally, **Appendix I** shows the overall candidates' performance in each topic in 2023 and **Appendix II** shows the comparison of candidates' overall performance in 2022 and 2023.

2.0 ANALYSIS OF CANDIDATES' RESPONSES IN EACH QUESTION

The candidates' overall performance in each question is categorized based on the percentage of candidates who scored 3.5 marks or more. The categories are 60 - 100, 35 - 59 and 0 - 34 per cent for good, average and weak performance respectively. Similarly, in the graphs or charts, the categories are coloured green, yellow and red for good, average and weak. The candidates' performance is explained based on the strengths and weaknesses observed in the candidates' responses.

2.1 Question 1: Calculating Devices

The question assessed the competence of the candidates in using a non-programmable scientific calculator to evaluate the mathematical expressions involving exponential and logarithmic terms, the integral of a function and the roots of the polynomial function. The question required the candidates to use a non – programmable scientific calculator to:

(a) *compute the value of* $\dfrac{67.9\times\sqrt[3]{68.53}}{\sqrt[4]{e^3\ln 2}}$ *correct to 5 significant figures.*

(b) *evaluate* $\int_0^1 e^{x^2}dx$ *correct to 4 decimal places.*

(c) *approximate the value(s) of* x *(correct to 3 decimal places) which satisfy the equation* $x^3+5x^2+3x-7=0$.

In this question, 31,039 (80.92%) candidates scored marks ranging from 3.5 to 10. Therefore, candidates' overall performance in this question was good. The following figure shows the percentage of candidates who scored low, average and high marks on this question.

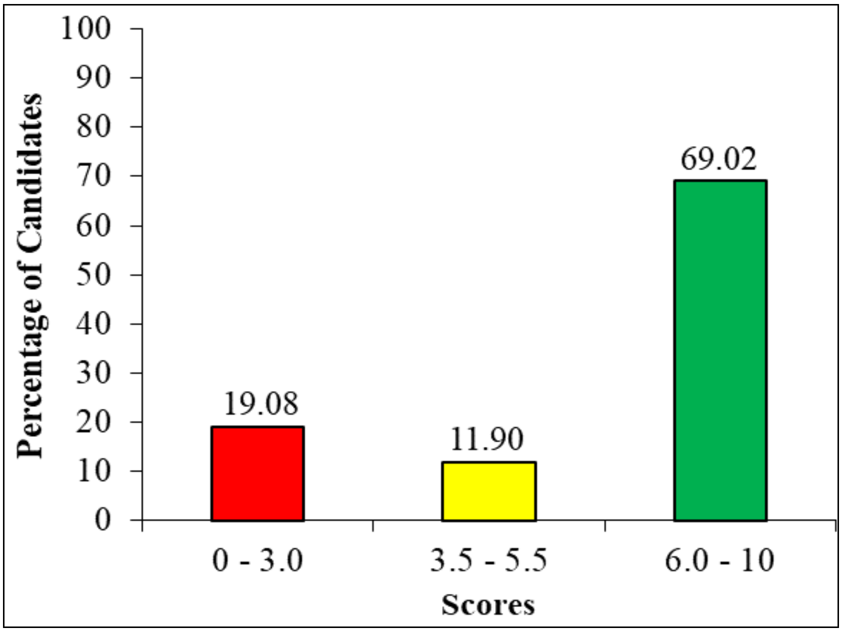


Figure 2: *The candidates' performance in Question 1*

In this question, 9,229 (24.06%) candidates correctly responded to all parts and thus, they scored 10 marks. In part (a), these candidates correctly calculated the value of $\frac{67.9\times\sqrt[3]{68.53}}{\sqrt[4]{e^3 \ln 2}}$ by inserting the expression into the calculator and they got 143.8476567. Then, they set the calculator to read five significant figures by pressing SHIFT followed by MODE, 7 and 5; and they resulted in 143.85. In part (b), the candidates reached the menu for integration $\int dx$ by pressing the SHIFT button followed by the alpha button.

Within the bracket, they inserted the expression e^x by pressing SHIFT + ln buttons followed by the button for powers $(\wedge)$ and finally ALPHA +) buttons for the variable *x*. Then, they inserted comma (using the comma button), 0 (lower limit), comma again, 1 (upper limit), closing brackets and finally pressed the button with equal sign and got 1.4627. Then, they set the calculator to read four decimal places by pressing SHIFT followed by MODE, 6 and 3; and got 1.4627. In part (c) the candidates correctly navigated to the menu for solving cubic equations by pressing MODE followed by 3. Then, they inserted 1, 5, 3 and -7 for *a*, *b*, *c* and *d* respectively. After pressing the button with an equal sign, they obtained $x_1 = 0.86666666$, $x_2 = -3.6555555$ and $x_3 = -2.2111111$. They finally set the calculator to read three decimal places and got the answers; $x_1 = 0.866$,

$x_2 = -3.655$ and $x_3 = -2.211$. Extract 1.1 illustrates correct responses to this question from one of the candidates.

1 (a)	143.85 (correct to 5 significant figures)
(b)	1.4627 (correct to 4 decimal places)
(c)	$x_1 = -3.655$ (3 decimal places)
	$x_2 = 0.866$ (3 decimal places)
	$x_3 = -2.211$ (correct to 3 decimal places)

Extract 1.1: A sample of the correct responses to Question 1.

In Extract 1.1, the candidate correctly evaluated the mathematical expressions involving exponential and logarithmic terms, the integral of a function and the roots of the polynomial function.

Despite the good performance, a number of candidates had incorrect responses to some parts of the question. Most candidates had incorrect answers in part (a) due to failure to set the calculator to read the fixed number of significant figures whereby 143.8476567 was commonly observed.

In part (b) some candidates did not adhere to the algorithm of inserting the components of integration into the calculator. Most of these candidates wrote upper limit and then lower limit (interchanged the position of upper and lower limits) and thus, ended up with -1.4627 instead of 1.4627. Other candidates misinterpreted e^{x^2} as $\left(e^x\right)^2$ which is equivalent to e^{2x} and therefore, they evaluated $\int_0^1 e^{2x}dx$ and got 3.1945.

Similar to part (b), in part (c) many candidates failed to set the calculator to read the fixed number of decimal places. Most candidates wrote the answers with more than 3 decimal places, particularly $x_1 = 0.86666666$, $x_2 = -3.6555555$ and $x_3 = -2.2111111$. It is also noted that several candidates incorrectly developed quadratic equations from the given cubic equation (see Extract 1.2). This approach indicates that they were able to navigate the menu of solving quadratic equations but not cubic equations, however, the steps for both menus are almost the same. Extract 1.2 is a sample response from one of the candidates who incorrectly responded to this question.

(c) given that

$x^3 + 5x^2 + 3x - 7 = 0$

by differentiate the equation respect with x

$3x^2 + 10x + 3 = 0$

on solving

The value of $x = -0.333$

Extract 1.2: A sample of the incorrect responses to part (c) of Question 1.

In Extract 1.2, the candidate incorrectly developed a quadratic equation from the given cubic equation by evaluating the first derivative of $x^3 + 5x^2 + 3x - 7 = 0$.

2.2 Question 2: Functions

The question assessed the competence of the candidates in defining the function, sketching the graph of a rational function and identifying its domain and range. The question stated as follows:

The function f *is defined as* $f(x) = \frac{a}{x} + b$ *such that* $f(2) = 2$ *and* $f(-1) = -1$.

(a) *Find the value of* a *and* b.

(b) *Sketch the graph of* f.

(c) *State the domain and range of* f.

The overall performance of the candidates in this question was average since 15,279 (39.83%) candidates got 3.5 marks or more. Figure 3 illustrates the percentage of the candidates who scored low, average or high marks.

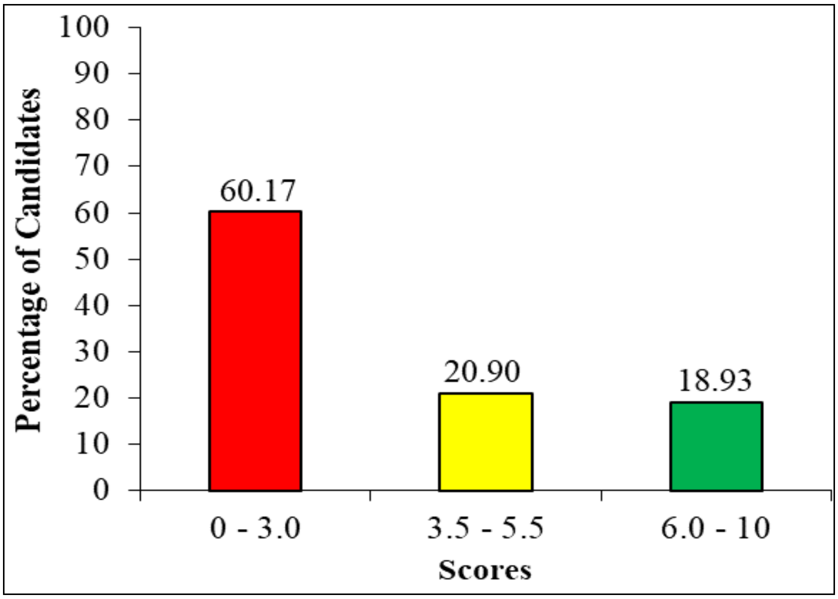
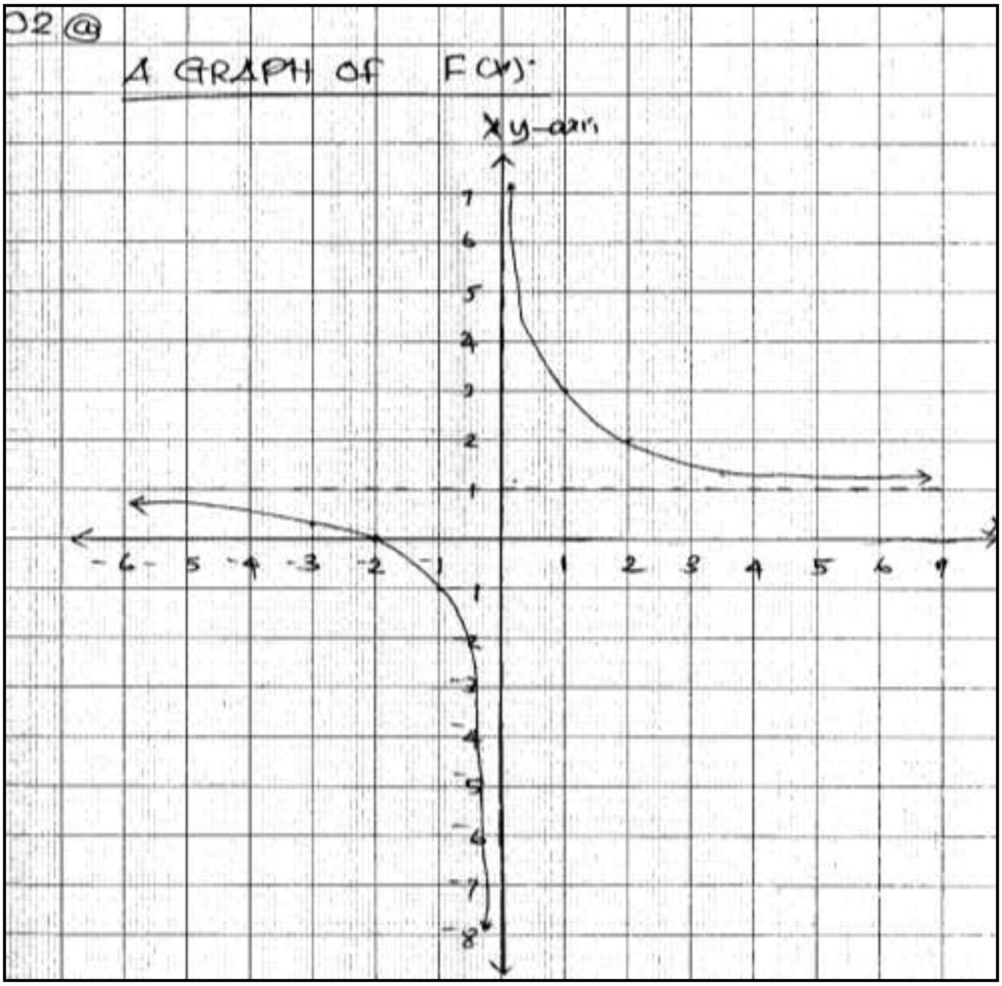


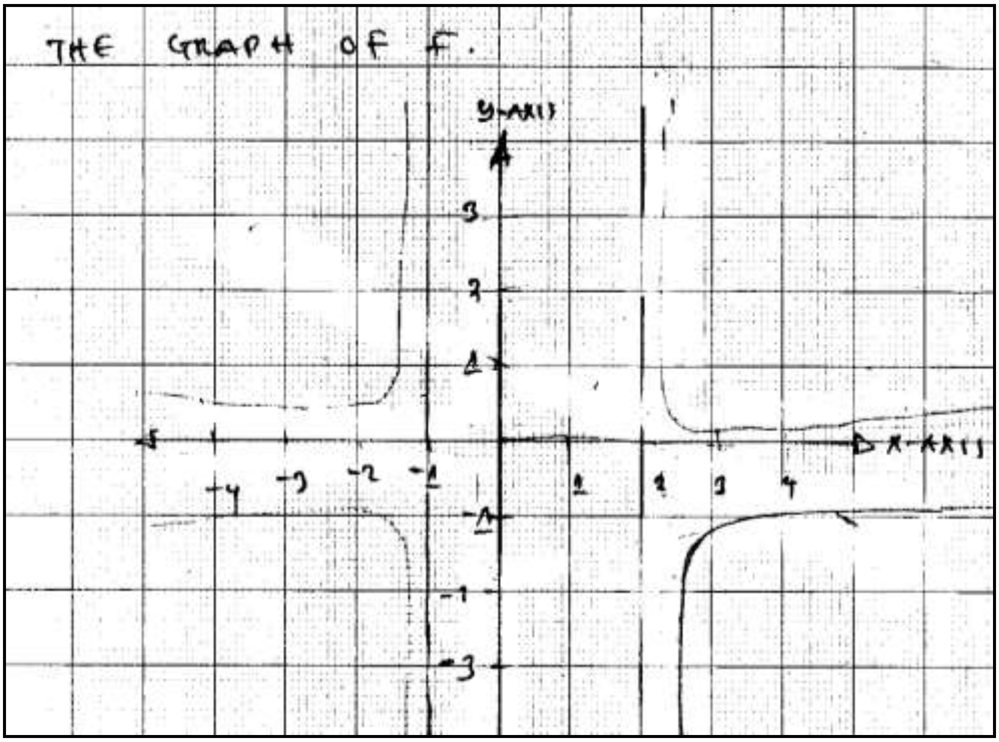
Figure 3: *The candidates' performance in Question 2*

Only 2,253 (5.87%) candidates correctly responded to all parts of this question. In part (a), the candidates were conversant with the definition of a function. With $f(x)=\frac{a}{x}+b$, they realized that $f(2)=2$ implies $\frac{a}{2}+b=2$ and $f(-1)=-1$ implies $\frac{a}{-1}+b=-1$. Therefore, they produced the equations $a+2b=4$ and $-a+b=-1$ and solved them simultaneously and got $a=2$ and $b=1$. In part (b), the candidates realized that $f(x)=\frac{2}{x}+1$, which is the rational function. Therefore, the candidates correctly determined that x intercept is -2 and the function has no y – intercept. They also correctly determined that the function has a vertical asymptote at $x=0$ and horizontal asymptote at $y=1$. Using these components and the properties of graphs of rational functions, they correctly sketched the graph of *f* (see Extract 2.1). Furthermore, these candidates realized that the graph includes all values along x – axis except 0 and therefore, correctly concluded that the domain includes all real numbers except 0. Similarly, they correctly realized that the graph excludes 1 along the y – axis and therefore, they concluded that the range includes all real numbers except 1. Extract 2.1 shows a correct graph presented by one of the candidates.



Extract 2.1: A sample of the correct responses to part (b) of Question 2.

On the other hand, the majority (60.17%) scored low marks. Most of these candidates failed to determine the correct values of *a* and *b* in part (a) due to failure to define the function or computational errors. For instance, some candidates correctly wrote $2=\frac{a}{2}+b$ and $-1=\frac{a}{-1}+b$, but they committed errors in simplifying the equations. Some of these candidates incorrectly simplified the equations and got $a+b=4$ and $a+b=1$ which imply that there are no real values for *a* and *b*. Other candidates also incorrectly solved the equations and got incorrect values $a=0$ and $b=-1$. These mistakes led to incorrect functions and consequently incorrect graphs. For example, the candidates who got $a=0$ and $b=-1$ resulted in constant function, $f(x)=-1$. Moreover, other candidates did not strive to find the values of x and y intercepts as well as horizontal and vertical asymptotes. Therefore, they drew graphs incorrectly reflecting the rational functions which is not related to the particular function, as seen in Extract 2.2.



Extract 2.2: A sample of the incorrect responses to part (b) of Question 2.

2.3 Question 3: Algebra

The question assessed the candidates' competence in calculating the sum of finite series for arithmetic progression and solving problems involving variations. The question comprised the following word problems:

(a) *The sum of the first three terms of an arithmetic progression is 3 and the sum of the first five terms is 20. Find the first term and the common difference.*

(b) *The volume of a cone varies jointly as its height and the square of its radius. The cone with radius 6cm and height of 10cm has a volume* $120\pi\ \text{cm}^3$. *Find the volume of the cone having a radius of 15cm and a height of 7cm.*

In this question, 17,221 (44.89%) candidates got 3.0 marks or less and 21,139 (55.11%) candidates scored 3.5 marks or more. Therefore, the overall performance of the candidates was average. Figure 3 shows the percentage of candidates who attained weak, average and good performance.

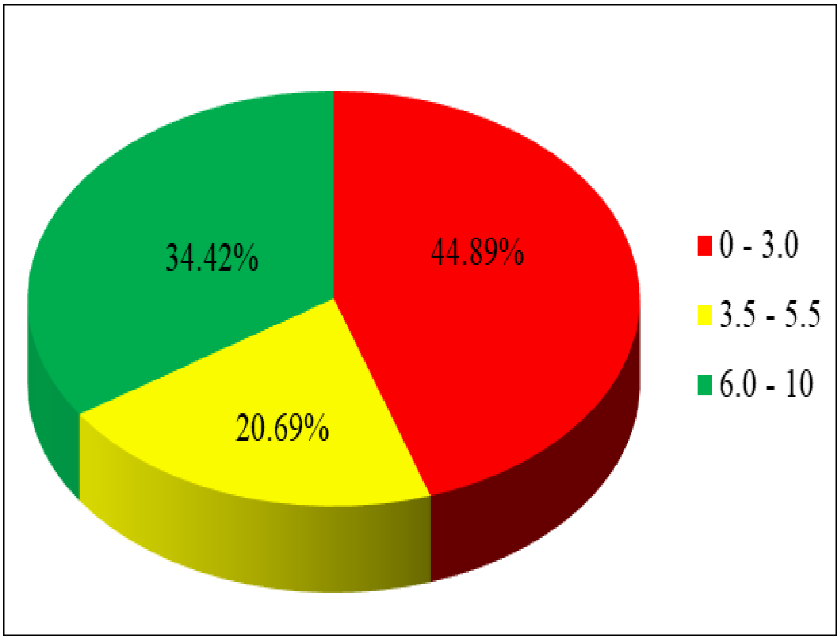


Figure 4: *The candidates' performance in Question 3*

A total of 8,439 (22.00%) candidates perfectly responded to the question. In part (a), the candidates correctly interpreted the word problem and hence, applied the formula for calculating the sum of the first *n* terms of Arithmetic Progression, $S_n = \frac{n}{2}(2A_1 + (n-1)d)$ where A_1 stands for the first term and d stands for the common difference. With this formula, they correctly developed the equations $A_1 + d = 1$ and $A_1 + 2d = 4$ and solved them simultaneously, hence obtained $d = 3$ and $A_1 = -2$.

In part (b), the candidates summarized the problem by writing the statement $V \alpha hr^2$ whereas *V*, *h* and *r* stand for volume, height and radius respectively. Thereafter, they introduced the constant k in $V \alpha hr^2$ and thus, the statement became $V = khr^2$. They further expressed k in terms of *V*, *h* and *r* and got $k = \frac{V}{hr^2}$ that implies $\frac{V_1}{h_1 r_1^2} = \frac{V_2}{h_2 r_2^2}$. Therefore, the candidates substituted $V_1 = 120\pi$, $h_1 = 10$, $r_1 = 6$, $h_2 = 7$ and $r_2 = 15$ into $\frac{V_1}{h_1 r_1^2} = \frac{V_2}{h_2 r_2^2}$ and correctly solved for the new volume (V_2) and got $V_2 = 525\pi \text{ cm}^2$. Extract 3.1 shows a sample response from one of the candidates who correctly responded to the question.

(b) Let volume of a cone be V.
Height of a cone be h
Radius of a cone be r^2.
According the relation
$V \propto h \propto r^2$.
$V \propto r^2 h$.
$V = K r^2 h$.
$k = \frac{V}{r^2 h}$

3(b) $K = \frac{120\pi \text{ cm}^3}{(6\text{cm})^2 (10\text{cm})}$
$k = \frac{\pi}{3}$
Then, The second case.
$V = kr^2 h$.
where,
$r = 15\text{cm}$
$h = 7\text{cm}$.
Then
$V = \frac{\pi}{3} \times (15\text{cm})^2 \times 7\text{cm}$
$V = 525\pi \text{ cm}^3$

Extract 3.1: A sample of the correct responses to part (b) of Question 3

In Extract 3.1, the candidate correctly computed the value of the constant *k* and hence developed the formula $V = \frac{1}{3}\pi h r^2$ and used it to calculate the new volume of the cone.

Despite the performance being good, 12,220 (31.86%) candidates got zero. In part (a), most candidates used inappropriate formulae (Extract 3.2). Apart from this, some candidates also applied the incorrect formulae. For instance, some candidates wrote $A_3 = A_1 - 2d$ and $A_5 = A_1 - 4d$ instead of $A_3 = A_1 + 2d$ and $A_5 = A_1 + 4d$. These candidates intended to use the formula for calculating n[th] term of Arithmetic Progression but failed to recall it correctly. Furthermore, some candidates correctly recalled the formula $S_n = \frac{n}{2}(2A_1 + (n-1)d)$, but they committed computation errors. For example, some candidates correctly obtained $A_1 + d = 1$ and $A_1 + 2d = 4$ but they got incorrect answers, $A_1 = -4$ and $d = 4$ instead of $A_1 = -2$ and $d = 3$.

In part (b), most candidates incorrectly interpreted the problem. Some candidates wrote $V\alpha\frac{h}{r^2}$ instead of $V\alpha hr^2$. As a result, these candidates got an incorrect statement $V=\frac{kh}{r^2}$ leading to incorrect values of the constant (k) and the new volume (V_2), commonly $k=1357.17$ and $V_2=42.23\,\text{cm}^3$. In addition, some candidates wrote $V=\frac{kh}{\sqrt{r}}$ indicating that the candidates misinterpreted the word 'square' as 'square root'. Extract 3.2 is a sample of incorrect responses to this question.

$$A_1 + 2d = 3 \quad \text{---(i)}$$
$$A_1 + 4d = 20 \quad \text{---(ii)}$$

By simultaneous Equation.

$$\begin{array}{l} 4\,\{ A_1 + 2d = 3 \\ 2\,\{ A_1 + 4d = 20 \end{array}$$
$$\begin{array}{l} 4A_1 + 8d = 12 \\ -\;2A_1 + 8d = 40 \end{array}$$
$$\frac{2A_1}{2} = \frac{-28}{2}$$
$$A_1 = -14.$$

Extract 3.2: A sample of the incorrect responses to part (a) of Question 3

In Extract 3.2, the candidate used the formula for calculating the nth term of an arithmetic progression instead of the formula for calculating the sum of the first n terms of an arithmetic progression.

2.4 Question 4: Differentiation

The question examined the ability of the candidates to determine derivatives of a function and apply differentiation in solving real-life problems. The question stated as follows:

(a) Find the first derivative for each of the following functions:

(i) $f(x)=\cos(2x+1)$.

(ii) $g(x)=\frac{x}{1+x^2}$.

(iii) $h(x)=3^x$.

(b) *The temperature* (T) *in* ${}^{0}C$ *of meat in a freezer after* t *hours is given by* $T = 70 - 12t + \dfrac{4}{t+1}$.

(i) *What is the temperature of the meat after 3 hours?*

(ii) *How fast is the temperature of the meat falling after 3 hours?*

A total of 11,474 (29.91%) candidates scored marks ranging from 3.5 to 10. Therefore, the candidates' overall performance in this question was weak. Figure 5 shows the percentage of candidates with low, average and high scores.

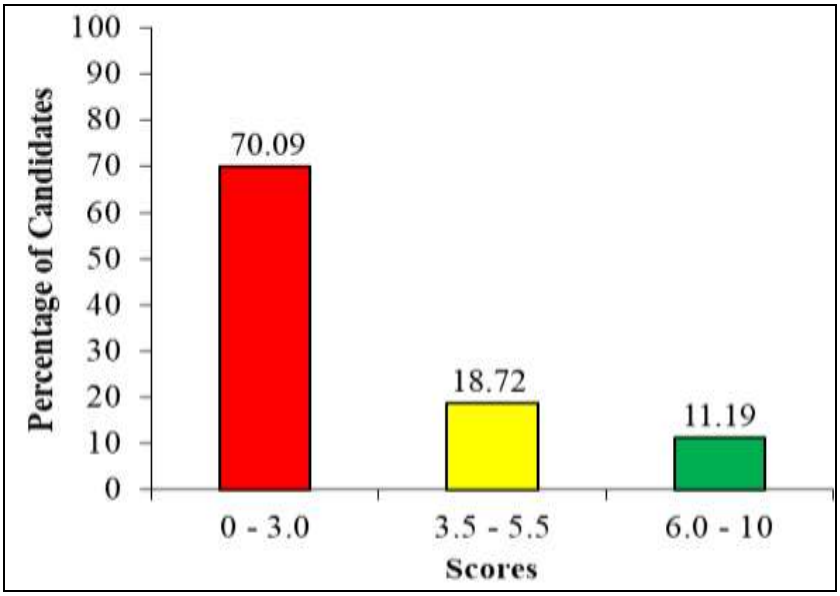


Figure 5: *The candidates' performance in Question 4*

As Figure 5 shows, 70.09 per cent of the candidates scored low marks. It was noted further that 17.62 per cent got zero. In part (a), the majority applied inappropriate techniques of differentiation. For example, some candidates applied the product rule in responding to part (a) (i) instead of the chain rule. They let $v = 2x+1$ and $u = \cos v$ and got the correct derivatives $\dfrac{dv}{dx} = 2$ and $\dfrac{du}{dx} = \sin u$. However, they substituted these components into $\dfrac{dy}{dx} = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$ and ended up with an incorrect answer, $f'(x) = \sin(2x+1) + \cos 2$. This question could be solved by applying the chain rule, $f'(x)=\frac{dv}{du}\times\frac{du}{dx}$. They were supposed to let $u=2x+1$ and $v=\cos u$ that give $\frac{du}{dx}=2$ and $\frac{dv}{du}=-\sin u$. Then, substitute these expressions into the formula $f'(x)=\frac{dv}{du}.\frac{du}{dx}$ and simplify the resulting expression to obtain $f'(x)=-2\sin(2x+1)$.

In part (a) (ii), several candidates incorrectly applied an inappropriate technique $\frac{dy}{dx}=ax^{a-1}$ and got $\frac{dy}{dx}=\frac{1}{1+2x}$. Instead, these candidates were supposed to use the quotient rule, $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vdu-udv}{v^2}$ whereas $v=1+x^2$, $\frac{dv}{dx}=2x$ and $u=x$, $\frac{du}{dx}=1$ that leads to $g'(x)=\frac{1-x^2}{\left(1+x^2\right)^2}$.

In part (a) (iii) many candidates wrote $\frac{dy}{dx}=x\times 3^{x-1}$ which indicates that they applied an inappropriate technique, $\frac{dy}{dx}=ax^{a-1}$. They were supposed to apply laws of logarithms to rewrite $y=3^x$ into $\ln y=x\ln 3$ and perform differentiation to get $h'(x)=3^x\ln 3$.

Few candidates responded to part (b) (i) incorrectly by substituting the value of t into some terms (not all) of the expression $70-12t+\frac{4}{t+1}$. For instance, some candidates substituted $t=3$ into the term $\frac{4}{t+1}$ only and hence, they ended up with $T=70-12t$. Few candidates converted 3 hours into minutes. Thus, they substituted $t=180$ into $T=70-12t+\frac{4}{t+1}$ and resulted in the incorrect answers, $T=-2089.98^0C$ in particular. The candidates were supposed to realize that the relation $T=70-12t+\frac{4}{t+1}$ has been conditioned to the time in hours (not otherwise). Therefore, they were supposed to substitute $t=3$ into the given relation, that is $T=70-12(3)+\frac{4}{3+1}$ and simplify it to get $35^\circ C$.

In part (b) (ii), most candidates got incorrect answer due to failure in performing differentiation. Many candidates got $\frac{dT}{dt} = -12 - \frac{4}{t^2}$ $\left(\text{instead of } \frac{dT}{dt} = -12 - \frac{4}{(t+1)^2}\right)$ from $T = 70 - 12t + \frac{4}{t+1}$, hence wrong rate, $\frac{dT}{dt} = -12.44^0 C/\text{hour}$. Some candidates incorrectly performed differentiation on $T = 70 - 12t + \frac{4}{t+1}$ and got $dT/_{dt} = -12t - t$ that resulted in $dT/_{dt} = -15^0 C/\text{hour}$. Extract 4.1 shows a sample of the incorrect responses of the candidates.

ii) soln

Given the time after 3hr:

$= 35°C$

Then $\frac{35°}{3} = 11.67°C\,hr^{-1}$.

$\therefore$ The rate at which the temperature of the meat falling after 3hrs

$= 11.67°C\,hr^{-1}$.

Extract 4.1: A sample of the incorrect responses to part (b) of Question 4.

In Extract 4.1, the candidate computed the rate of change of temperature by dividing the temperature of the meat $(35^\circ C)$ by the duration of 3 hours.

About 11.19 per cent, equivalent to 4,291 candidates, scored high marks on this question. Among them, 617 (1.61%) candidates obtained full marks. In part (a) (i), these candidates correctly applied the chain rule. They let $u = 2x+1$, $\frac{du}{dx} = 2$ and $v = \cos u$, $\frac{dv}{du} = -\sin u$. Then, they replaced $\frac{du}{dx}$ and $\frac{dv}{du}$ in the $f'(x) = \frac{dv}{du} \cdot \frac{du}{dx}$ with 2 and $-\sin u$ and correctly simplified the resulting expression and got $f'(x) = -2\sin(2x+1)$. In part (a) (ii) these candidates correctly applied the quotient rule, $g'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ where

$v = 1 + x^2$, $u = x$, $\frac{dv}{dx} = 2x$ and $\frac{du}{dx} = 1$, that was correctly simplified to get $g'(x) = \frac{1 - x^2}{1 + 2x^2 + x^4}$. In part (a) (iii), the candidates correctly applied the concept of natural logarithm on $h(x) = 3^x$ and got $\ln|h(x)| = x\ln 3$. Then, they performed differentiation and simplification on $\ln|h(x)| = x\ln 3$ and obtained $h'(x) = 3^x \ln 3$. In part (b), these candidates were conversant with the application of differentiation in solving problems related to the rate of change as Extract 4.2 shows.

4(b) (ii) Given $T = 70 - 12t + \frac{4}{t+1}$.

Differentiate with respect to t

$$\frac{dT}{dt} = \frac{d}{dt}\left(70 - 12t + \frac{4}{t+1}\right)$$

$$\frac{dT}{dt} = -12 + \frac{d}{dt}\left(\frac{4}{t+1}\right).$$

$$\frac{dT}{dt} = -12 + \left(\frac{0(t+1) - 4(1)}{(t+1)^2}\right)$$

$$\frac{dT}{dt} = -12 + \left(\frac{-4}{(t+1)^2}\right)$$

$t = 3$ hours.

$$\frac{dT}{dt} = -12 + \left(\frac{-4}{(3+1)^2}\right) = -12.25°C \text{ per hour}$$

Extract 4.2: A sample of correct responses to part (b) of Question 4.

In Extract 4.2, the candidate correctly realized that the expression of rate of change equals the derivative of the given function.

2.5 Question 5: Integration

The question assessed the candidates' competence in evaluating the definite integrals of the polynomial functions and using the substitution technique to integrate polynomial functions. The question consisted of the following parts:

(a) *Given that* $\int_1^5 h(x)dx = 4$.

(i) *Evaluate* $\int_1^5 (h(x)+3)dx$.

(ii) *Find the value of* k *if* $\int_1^5 (h(x)+kx)dx = 28$.

(b) *Find* $\int t(1+5t)^7\,dt$.

The performance of the candidates in this question was generally weak (observe Figure 6).

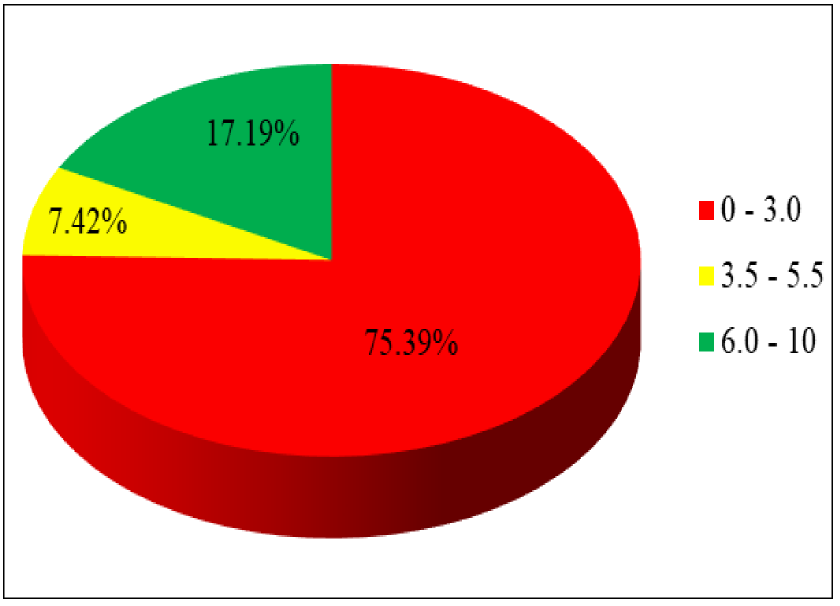


Figure 6: *The candidates' performance in Question 5*

As Figure 6 shows, 75.39 per cent, equivalent to 28,919 candidates, scored low marks and among them, 19,404 (50.58%) candidates got zero. In part (a) (i), the common weakness was the failure to apply the property $\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$. Many candidates used an incorrect approach by replacing $h(x)$ in $\int_1^5 (h(x)+3)dx$ with 4 and got $\int_1^5 (7)dx$ that leads to 28. Other candidates correctly rewrote $\int_1^5 (h(x)+3)dx$ in the form

$\int_1^5 h(x)dx + 3\int_1^5 dx$ and replaced $\int_1^5 h(x)dx$ with 4 to get $4+3\int_1^5 dx$. However, they failed to integrate $3\int_1^5 dx$ as they got 3 and consequently $\int_1^5 (h(x)+3)dx = 7$. Indeed, $3\int_1^5 dx$ equals $[3x]_1^5$ and results in 12, therefore $\int_1^5 (h(x)+3)dx$ becomes $4+12$ which is equivalent to 16. Similar mistakes were observed in part (a) (ii), as Extract 5.1 illustrates.

In part (b), most of these candidates realized that the appropriate technique for determining the integral of $t(1+5t)^7$ with respect to t is substitution. However, they failed to rewrite the expression $t(1+5t)^7$ into a form which could be easy to integrate. Most of these candidates ended up with an expression involving more than one variable. For instance, some candidates correctly obtained $dt = \frac{du}{5}$ after letting $u = 1+5t$, but they did not make t the subject of $u = 1+5t$. Therefore, they replaced $1+5t$ and dt in the $\int t(1+5t)^7 dt$ with u and $\frac{du}{5}$ while leaving t unchanged. Thus, $\int t(1+5t)^7 dt$ became $\int tu^7 \frac{du}{5}$, which involves two variables, t and u. These candidates were supposed to produce both $t = \frac{u-1}{5}$ and $dt = \frac{du}{5}$ from $u = 1+5t$ that would enable them to change $\int t(1+5t)^7 dt$ into $\int \frac{u^8 - u^7}{25} du$. Instead, they worked on $\int tu^7 \frac{du}{5}$ and got incorrect answers, including $\frac{1}{5}t\frac{u^8}{8} + c$ that resulted in $\frac{t}{40}(1+5t)^8 + c$ after substitutions and simplifications. In addition, some of these candidates incorrectly worked on $t(1+5t)^7$ and produced $(t+5t^2)^7$. They also performed incomplete substitutions resulting in the integrand involving two variables, particularly $\int \frac{u^7}{1+10t} du$. Moreover, a significant number of these candidates incorrectly simplified $t(1+5t)^7$ into t^7+5t^{14}. As a result, they worked on $\int t^7 dt + 5\int t^{14} dt$ and obtained incorrect answers for $\int (t+5t^2)^7 dt$. Extract 5.1 is an incorrect responses from one of the candidates

ii) Soln

$$\int_1^5 \left(h(x) + kx \right) dx = 28$$

$$\int_1^5 (4 + kx)\, dx = 28$$

$$\left[4x + \frac{kx^2}{2} \right]_1^5 = 28$$

$$\left[4(5) + \frac{k(5)^2}{2} \right] - \left[4(1) + \frac{k(1)^2}{2} \right] = 28$$

$$\left[\frac{20}{1} + \frac{25k}{2} \right] - \left[\frac{4}{1} + \frac{k}{2} \right] = 28$$

$$\left[\frac{40+25k}{2} \right] - \left[\frac{8+k}{2} \right] = 28$$

$$\frac{40-25k-8-k}{2} = \frac{28}{1}$$

$$40-25k-8-k = 56$$

$$32-26k = 56$$

$$-26k = 56-32$$

$$\frac{\cancel{-26}k}{\cancel{-26}} = \frac{24}{-26}$$

$$k = -0.923$$

Extract 5.1: A sample of the incorrect responses to part (a) of Question 5.

In Extract 5.1, the candidate incorrectly assumed that $h(x)=4$ instead of $\int_1^5 h(x)\,dx = 4$.

A total of 6,596 (17.19%) candidates scored high marks, of which, 1.96 per cent obtained full marks. In part (a), these candidates were knowledgeable about the property $\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$. In part (a) (i), these candidates correctly rewrote $\int_1^5 (h(x)+3)dx$ in the form $\int_1^5 h(x)dx + 3\int_1^5 dx$ and replaced $\int_1^5 h(x)\,dx$ with 4 resulting in $4+3\int_1^5 dx$. They then correctly evaluated $3\int_1^5 dx$ and got $[3x]_1^5$ that gives 12 and therefore, $\int_1^5 (h(x)+3)\,dx = 4+12 = 16$. Likewise, in part (a) (ii), these candidates correctly evaluated $\int_1^5 (h(x)+kx)dx$ and got $4+12k$. Then, they equated $4+12k$ to 28 and correctly solved for k to get $k=2$.

In part (b), some candidates demonstrated competence in applying substitution technique to evaluate the indefinite integrals. They let $u = 1+5t$ and correctly produced $t = \frac{u-1}{5}$ and $dt = \frac{du}{5}$. They then performed appropriate substitutions that changed $\int t(1+5t)^7\,dt$ into $\int \frac{u^8 - u^7}{25} du$, which gives $\frac{1}{25}u^8\left[\frac{8u-9}{72}\right]+C$. Finally, they replaced u in $\frac{1}{25}u^8\left[\frac{8u-9}{72}\right]+C$ with $1+5t$ to get the integral in terms of t, $\frac{1}{1800}(1+5t)^8[8(1+5t)-9]+C$. Extract 5.2 shows candidate's correct responses to this question.

5(b) Given

$$\int t\,(1+5t)^7\,dt$$

Soln

Let

$u = 1+5t$ so that

$$\frac{du}{dt} = 5$$

$$dt = \frac{du}{5}$$

Then;

$$t = \frac{u-1}{5}$$

Then;

$$= \int \left(\frac{u-1}{5}\right) u^7 \,\frac{du}{5}$$

$$= \frac{1}{25}\int u^7(u-1)\,du$$

$$= \frac{1}{25}\int (u^8 - u^7)\,du$$

5(b)

$$= \frac{1}{25}\left(\frac{u^9}{9} - \frac{u^8}{8}\right) + C$$

$$= \frac{1}{25}\left(\frac{(1+5t)^9}{9} - \frac{(1+5t)^8}{8}\right) + C$$

$$= \frac{(1+5t)^8}{25}\left(\frac{1+5t}{9} - \frac{1}{8}\right) + C$$

$$= \frac{(1+5t)^8}{25}\left(\frac{8+40t-9}{72}\right) + C$$

$$= \frac{(1+5t)^8}{25}\left(\frac{40t-1}{72}\right) + C$$

$$= \frac{(1+5t)^8(40t-1)}{1800} + C$$

Extract 5.2: A sample of correct responses to part (b) of Question 5

In Extract 5.2, the candidate correctly applied the substitution technique.

2.6 Question 6: Statistics

The question required the candidates to summarize the data, graphically present the data and calculate the median and percentiles. The question was as follows:

Consider the following data;

28	*46*	*62*	*8*	*30*	*21*	*60*	*40*	*10*	*13*	*31*	*47*	*45*
31	*25*	*15*	*55*	*18*	*34*	*46*	*20*	*30*	*18*	*9*	*38*	*42*
32	*52*	*32*	*67*	*9*	*70*	*31*	*29*	*50*	*25*	*18*	*25*	*63*
42	*48*	*47*	*30*	*21*	*35*	*54*	*45*	*8*	*39*	*54*	*61*	*63*
12	*50*	*38*	*24*	*45*	*11*	*20*	*47*	*55*	*43*	*46*	*53*	*25*

(a) *Construct a frequency distribution table using the intervals $0 - 9, 10 - 19, \ldots, \ldots$*

(b) *Draw a histogram and use it to estimate the mode correct to 2 decimal places.*

(c) *Calculate:*

(i) *the median (correct to 3 decimal places)*

(ii) *the 70th percentile (correct to 3 decimal places)*

A total of 36,105 (94.12%) candidates scored 3.5 marks or more. Thus, the overall performance of the candidates in this question was good. The following graph shows the percentage of candidates who got low, average and high marks.

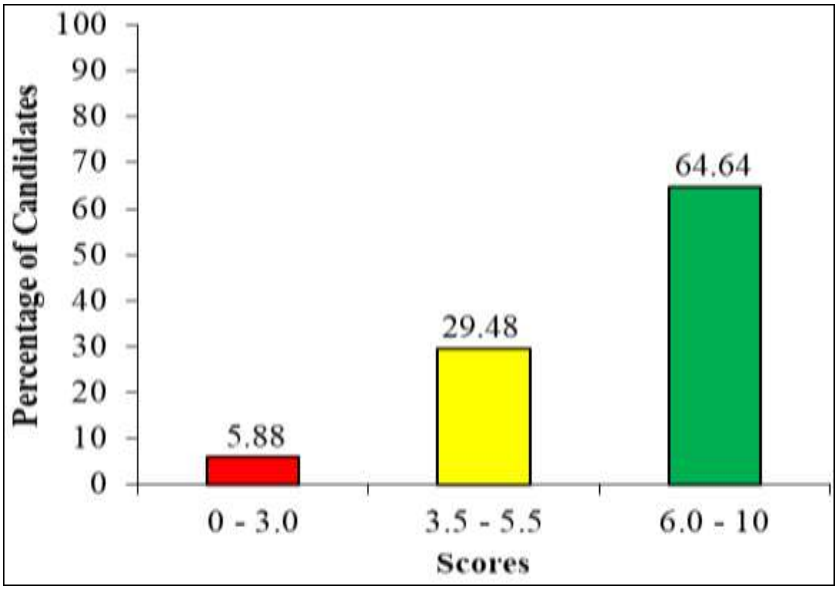
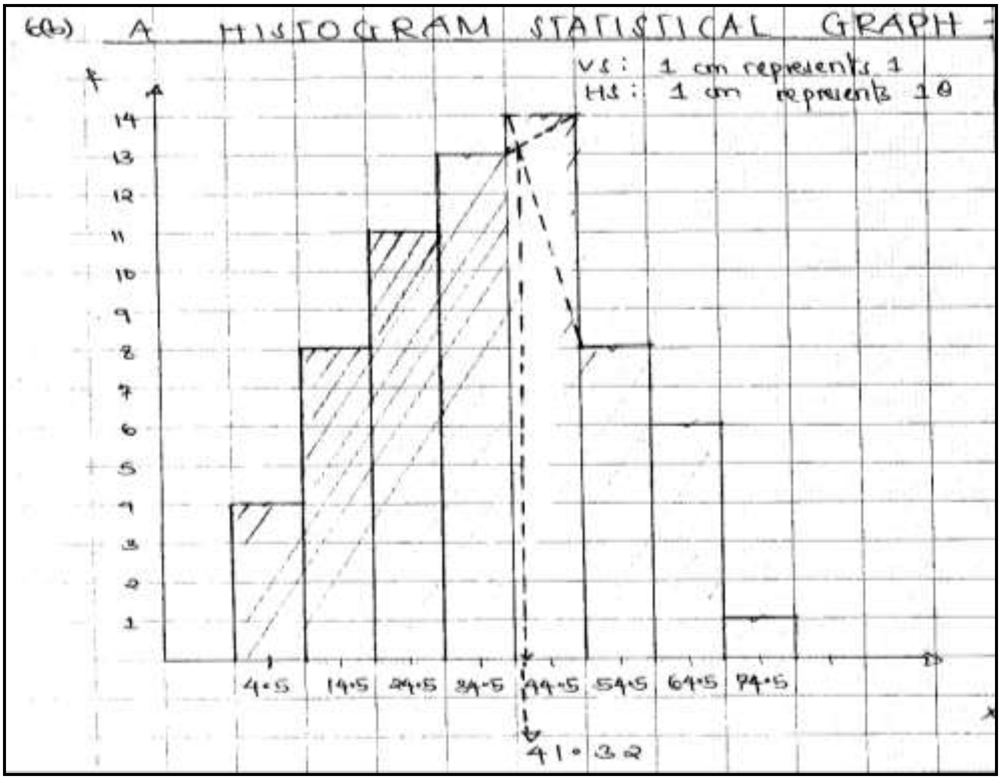


Figure 7: *The candidates' performance in Question 6*

As Figure 7 shows, 64.64 per cent, equivalent to 24,796 candidates, scored high marks (6.0 – 10) whereby 3,367 (8.78%) candidates got full marks. In part (a), most candidates correctly identified the classes and respective frequencies. In part (b), they correctly computed the classmark of each class and used it to draw a histogram (see Extract 6.1).

In part (c), the candidates correctly determined the median class (30 – 39) as it contains the cumulative frequency that marks half (33) of the total frequency. Then, they applied the formula $Median = L + \left(\frac{\frac{N}{2} - fb}{fw} \right) i$ where N stands for total frequency for all classes (65), fb for total frequency of all classes with less value than the median class (23) and fw for frequency in the median class (13). They also correctly performed computations and got $median = 36.808$. Similarly, these candidates correctly applied the formula for calculating the nth percentile. Extracts 6.1 and 6.2 illustrate correct responses to this question.



Extract 6.1: A sample of correct responses to part (b) of Question 6.

In Extract 6.1, the candidate correctly estimated the mode using the histogram.

6(c)(i) From

$$\text{Median} = L_m + \left(\frac{\frac{N}{2} - f_b}{f_w}\right) c$$

$$= 29.5 + \left(\frac{\frac{65}{2} - 23}{13}\right) 10$$

$$= 36.808$$

$\therefore$ Median $= 36.808$ (to 3 dps)

(ii). 70^{th} percentile.

Soln.

From,

$$P_{70} = L_{70} + \left(\frac{\frac{70}{100}N - f_b}{f_w}\right) C$$

$$= 39.5 + \left(\frac{\frac{70}{100}(65) - 36}{14}\right) 10$$

$$= 46.286$$

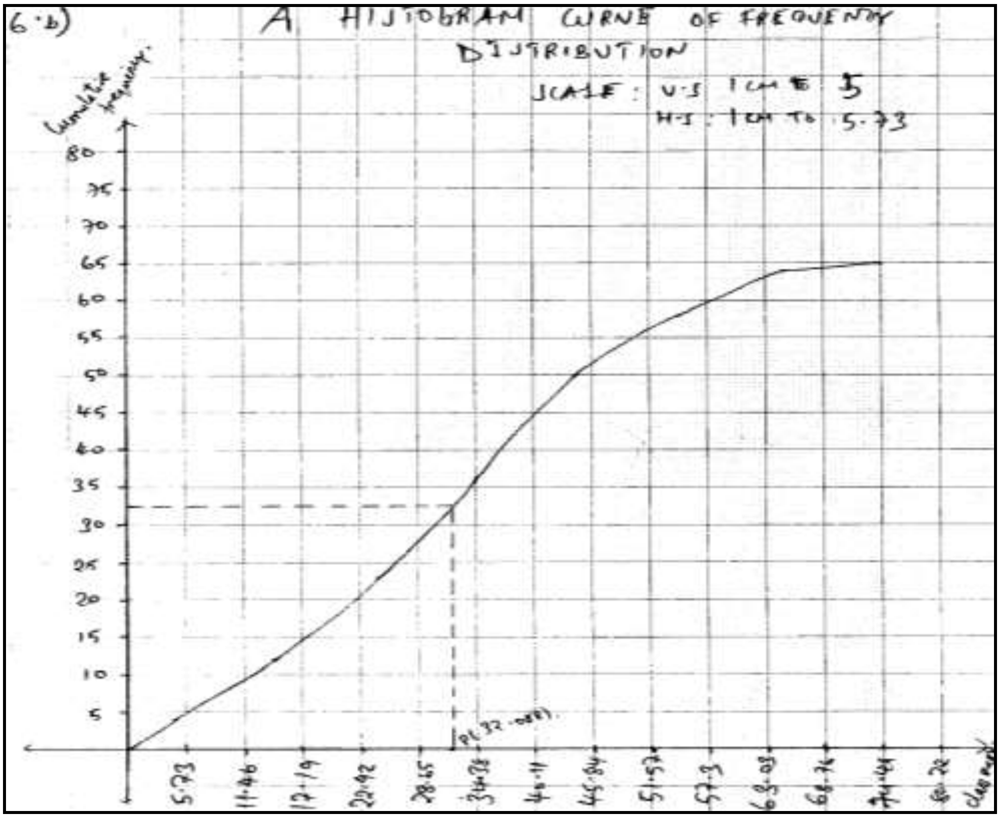
$\therefore$ 70^{th} percentile $= 46.286$ (to 3 dps

Extract 6.2: A sample of correct responses to part (c) of Question 6.

In Extract 6.2, the candidate correctly identified the class of 70^{th} percentile (40 – 49) and therefore, he/she correctly calculated the 70^{th} percentile.

In spite of good performance in this question, a considerable number of candidates (5.88%) got 3.0 marks or less. In part (a), most candidates failed to get the correct frequency for some classes. There were also candidates who wrote incorrect classes, such as 0.5 – 10.5, 10.5 – 20.5, . . . instead of 0 – 9, 10 – 19 , …. As a result, these candidates drew incorrect histograms in part (b). Moreover, some candidates drew separate rectangular bars and hence, bar graph (not a histogram). Therefore, these candidates got the value of mode which is not in the agreed range of 40.43 – 41.43.

In part (c), many candidates used inappropriate or incorrect formulae for a particular statistical measure. For instance, in part (b) (i), some candidates applied the formula for calculating mode $\left(\mathrm{M} = L_1 + \left(\frac{t_1}{t_1 + t_2}\right)i\right)$ while other candidates incorrectly recalled the formula for calculating median, $\text{Median} = L_1 + \left(\frac{\frac{N}{2} + n_b}{n_w}\right)i$ instead of $\text{Median} = L_1 + \left(\frac{\frac{N}{2} - n_b}{n_w}\right)i$. Apart from these mistakes, some candidates also failed to determine the 70^{th} percentile class and hence, got incorrect answers. For example, some candidates wrote $L_1 = 69.5$, $n_b = 64$, $n_w = 1$ and $c = 10$ indicating that they considered an incorrect class, that is, 70 – 79 instead of 40 - 49. Extract 6.3 shows incorrect responses from one of these candidates.



Extract 6.3: A sample of incorrect responses to part (b) of Question 6.

In Extract 6.3, the candidate drew a cumulative frequency curve (however, the curve is also incorrect) instead of a histogram.

2.7 Question 7: Probability

The question is based on the application of combinations in daily life situations and the probability of combined events. This question consisted of the following word problems:

(a) *A certain family consists of mother, father and their ten children. The family is invited to send a group of four representatives to a wedding. In how many ways can the group be formed if it must include both parents?*

(b) *A fair coin is tossed three times, using tree diagram; find the probability of obtaining exactly two heads.*

A total of 28,494 (74.28%) candidates obtained marks ranging from 3.5 to 10, of which 1.24 per cent obtained full marks. The following figure is a summary of the candidates' performance in this question.

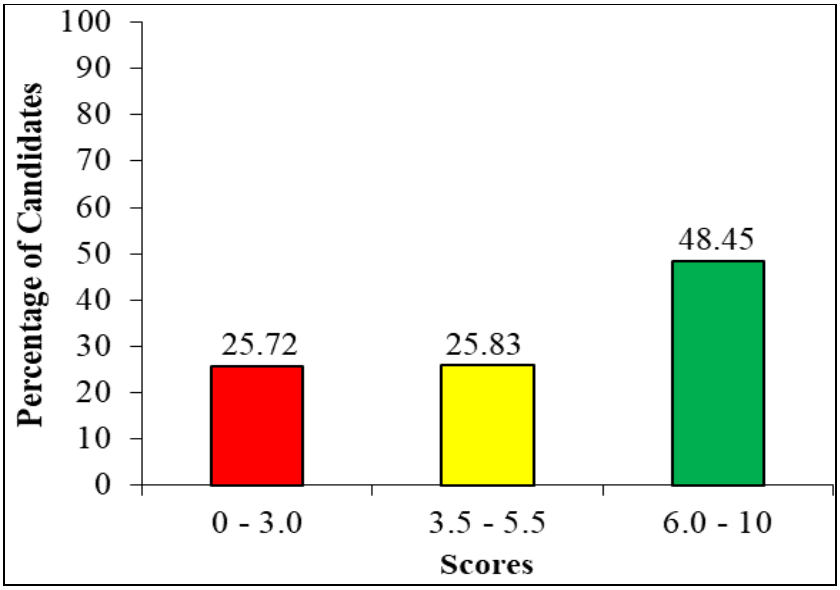
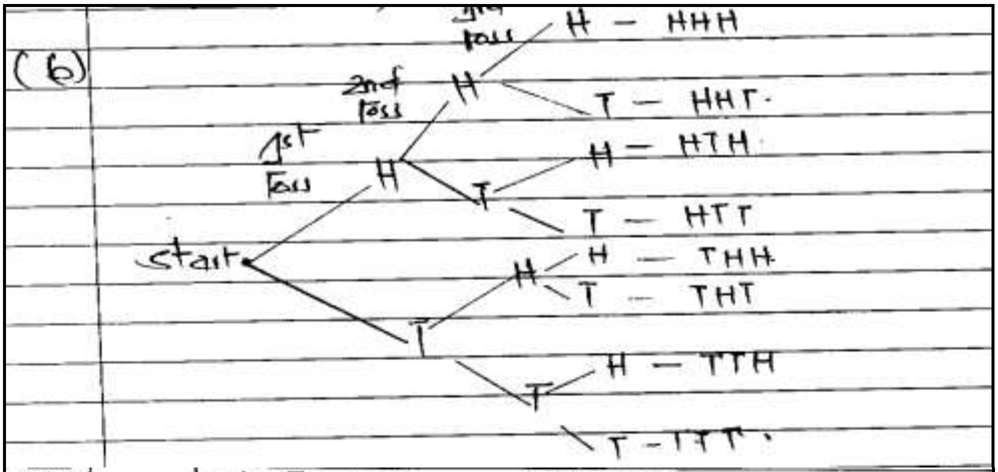


Figure 7: *The candidates' performance in Question 6*

In part (a), competent candidates were aware that the problem is solved by applying the knowledge of combinations. They realized that out of 4 representatives, father and mother were mandatory and hence the other 2 representatives are to be chosen from 10 children. Therefore, they calculated the number of ways (combinations) of forming a group of 2 objects from 10 unlike objects by evaluating ${}^{10}C_2$ that gives 45.

In part (b), the candidates drew a tree diagram correctly and applied the appropriate formula for calculating the probability of an event as Extract 7.1 illustrates.



7 (b) ~~total~~ Total outcomes $n(S) = 8$

exactly two heads, $n(E) = 3$.

from

$P(E) = n(E)/n(S)$.

$= 3/8$.

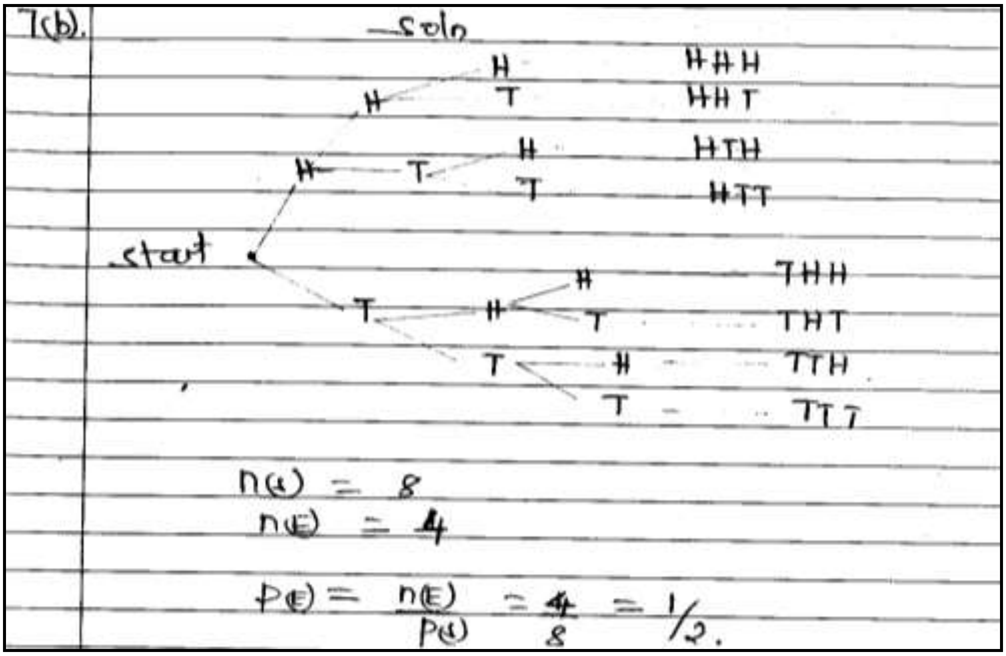
∴ Probability of getting exactly two heads $= 3/8$

Extract 7.1: A sample of correct responses to part (b) of Question 7.

In Extract 7.1, the candidate correctly identified the number of sample space and the event from the tree diagram.

Despite the good performance, 9,866 (25.72%) candidates scored low marks. Some of these candidates did not understand that the problem in part (a) is related to the concept of combinations. Some candidates computed the product of 12 and 4, hence concluded that there were 48 ways, while others divided 12 by 4 and got 3 ways. In addition, a significant number of candidates misinterpreted the problem. Most of these candidates assumed that all 4 representatives were not predetermined. They did not realize that the 2 representatives were conditioned to be father and mother. Therefore, these candidates evaluated ${}^{12}C_4$ (instead of ${}^{10}C_2$) and got incorrect answers such as 495 ways. Moreover, some candidates related the problem to the concept of permutations. They evaluated ${}^{12}P_4$ and ended up with an incorrect answer, 90 ways.

In part (b), most candidates presented a correct tree diagram, but they failed to identify the event. For example, some candidates wrote $n(E)=7$ and $n(S)=8$ resulting in $P(E)=\frac{7}{8}$. Extract 7.2 shows incorrect responses from one of the candidates.


7(b). Soln

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

$n(S) = 8$
$n(E) = 4$

$P(E) = \frac{n(E)}{P(S)} = \frac{4}{8} = 1/2.$

Extract 7.2: A sample of the incorrect responses to part (b) of Question 7.

In Extract 7.2, the candidate incorrectly identified the number of the event as 4 instead of 3.

2.8 Question 8: Trigonometry

The question tested the competence of candidates in applying the compound angle formulae in solving trigonometric problems. It included the following parts:

(a) *Prove that* $xy=\sin^2 A-\sin^2 B$ *whereby* $x=\sin(A+B)$ *and* $y=\sin(A+B)$.

(b) *Solve the following equations:*

(i) $2\sin^2\theta-3\cos\theta=3$.

(ii) $\sqrt{2}\cos\theta-\sin 2\theta$ *for* $0^0\le\theta\le 360^0$.

The overall performance of the candidates in this question was weak. About 78.90 per cent of the candidates scored low marks, while 21.10 per cent obtained average or high marks, as Figure 9 shows.

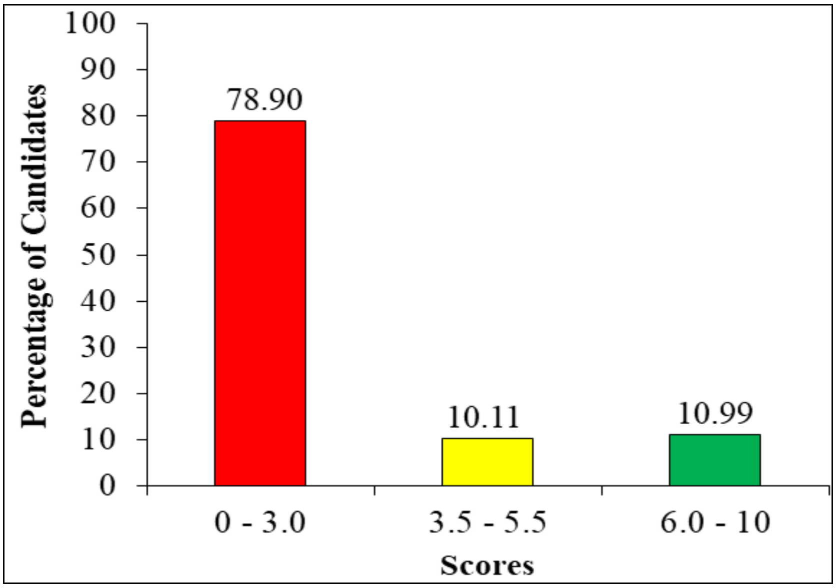


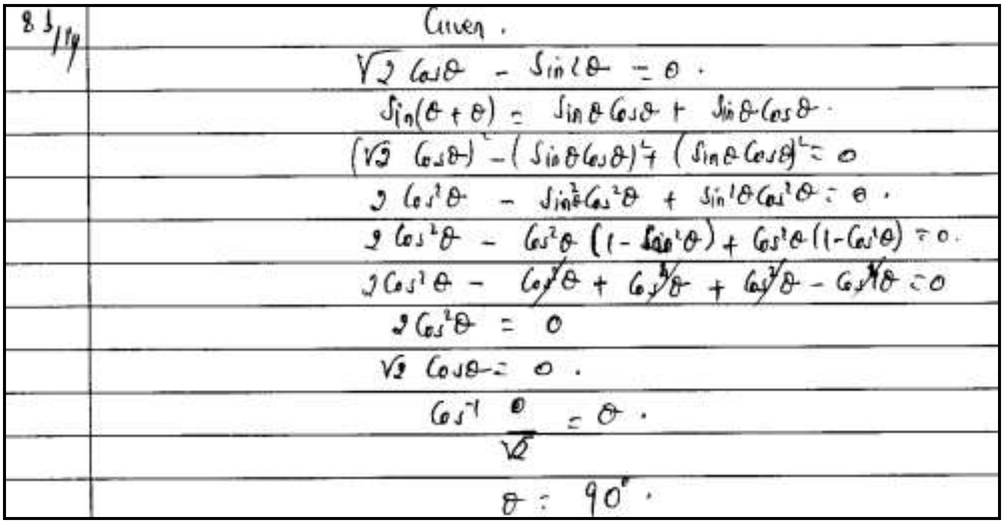
Figure 9: *The candidates' performance in question 8*

Moreover, 18,143 (47.30%) candidates got zero. In part (a), the majority incorrectly recalled the compound angle formulae. For instance, some candidates expanded $\sin(A-B)$ and $\sin(A+B)$ as $\sin A - \sin B$ and $\sin A + \sin B$ respectively. Though these expansions result in the given statement $xy = \sin^2 A - \sin^2 B$, they are mathematically incorrect. The correct expansions are $\sin(A-B) = \sin A\cos B - \cos A\sin B$ and $\sin(A+B) = \sin A\cos B + \cos A\sin B$. Other candidates wrote incorrect identities, particularly $\sin(A+B) = \sin(A\times B)$ and $\sin(A-B) = \sin\left(\frac{A}{B}\right)$. Therefore, they incorrectly maneuvered $\sin(A+B)\sin(A-B)$ and got the incorrect answers, including $\sin(A\times B)\sin^2\left(\frac{A}{B}\right)$ instead of $\sin^2 A - \sin^2 B$.

In part (b) (i), most of these candidates applied incorrect identities. For example, some candidates wrote $\sin^2\theta = 2(\sin\theta\cos\theta)$ instead of $\sin^2\theta = 1 - \cos^2\theta$. Therefore, they changed the equation into $2(\sin\theta\cos\theta) - 3\cos\theta = 3$ that gives the answers which are different from

120^0, 180^0 or 240^0. Some other candidates wrote $2\sin^2\theta - 3(1-\sin\theta) = 3$ indicating that they assumed $\cos\theta = 1-\sin\theta$.

Likewise, in part (b) (ii), most candidates assumed that $\sin 2\theta = \sin^2\theta + \cos^2\theta$ instead of $\sin 2\theta = 2\cos\theta\sin\theta$. Therefore, they changed the equation $\sqrt{2}\cos\theta - \sin 2\theta = 0$ into $\sqrt{2}\cos\theta - (\cos^2\theta + \sin^2\theta) = 0$. Some candidates correctly recalled the double angle formula for sine, but committed computational errors, as Extract 8.1 shows.



8.b)(ii)

Given.

$\sqrt{2}\cos\theta - \sin 2\theta = 0.$

$\sin(\theta+\theta) = \sin\theta\cos\theta + \sin\theta\cos\theta.$

$(\sqrt{2}\cos\theta)^2 - (\sin\theta\cos\theta)^2 + (\sin\theta\cos\theta)^2 = 0$

$2\cos^2\theta - \sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta = 0.$

$2\cos^2\theta - \cos^2\theta(1-\sin^2\theta) + \cos^2\theta(1-\cos^2\theta) = 0.$

$2\cos^2\theta - \cancel{\cos^2\theta} + \cancel{\cos^4\theta} + \cancel{\cos^2\theta} - \cancel{\cos^4\theta} = 0$

$2\cos^2\theta = 0$

$\sqrt{2}\cos\theta = 0.$

$\cos^{-1}\frac{0}{\sqrt{2}} = \theta.$

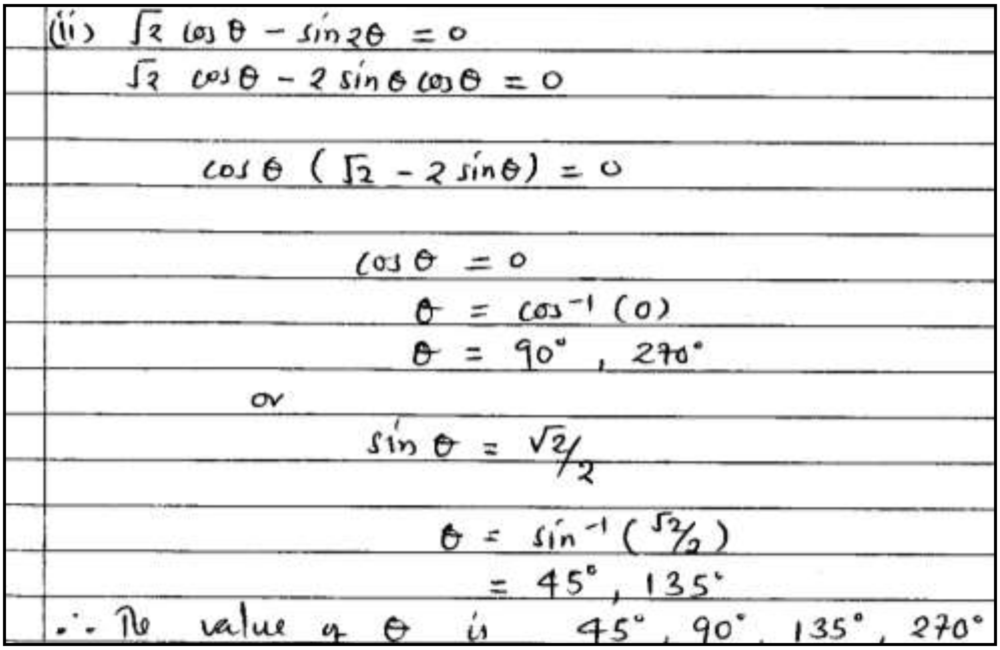
$\theta = 90^\circ.$

Extract 8.1: A sample of incorrect responses to part (b) of Question 8

In Extract 8.1, the candidate incorrectly worked on the equation by squaring each term separately.

Although the majority of candidates got low marks, about 0.80 per cent of the candidates obtained full marks. In part (a) these candidates correctly recalled and applied the compound angle formula for $\sin(A+B)$ and $\sin(A-B)$ resulting in $x = \sin A\cos B + \sin B\cos A$ and $y = \sin A\cos B - \sin B\cos A$. Thereafter, they correctly manoeuvered the product of $\sin A\cos B + \sin B\cos A$ and $\sin A\cos B - \sin B\cos A$ ending up getting $xy = \sin^2 A - \sin^2 B$.

In part (b) (i) these candidates applied the appropriate identity $\cos^2\theta + \sin^2\theta = 1$, from which they got $\sin^2\theta = 1-\cos^2\theta$. Therefore, they replaced $\sin^2\theta$ in $2\sin^2\theta - 3\cos\theta = 3$ with $1-\cos^2\theta$ and correctly simplified it into $2\cos^2\theta + 3\cos\theta + 1 = 0$ which involves $\cos\theta$ only. The equation was then solved and gave $\cos\theta = -1$ and $\cos\theta = -\frac{1}{2}$ that resulted in $\theta = 120°, 240°$ and $\theta = 180°$ respectively. Therefore, they correctly rearranged the angles and wrote $\theta = 120°, 180°, 240°$. In part (b) (ii), the candidates correctly applied the double angle formula for sine. They replaced $\sin 2\theta$ in $\sqrt{2}\cos\theta - \sin 2\theta = 0$ with $2\sin\theta\cos\theta$ and got $\sqrt{2}\cos\theta - 2\sin\theta\cos\theta = 0$. Then, they solved the equation by factorization to obtain $\cos\theta = 0$ and $\sin\theta = \frac{1}{\sqrt{2}}$, hence $\theta = 45°, 90°, 135°, 270°$. Extract 8.2 shows a sample of correct responses from one of the candidates.



(ii) $\sqrt{2}\cos\theta - \sin 2\theta = 0$

$\sqrt{2}\cos\theta - 2\sin\theta\cos\theta = 0$

$\cos\theta(\sqrt{2} - 2\sin\theta) = 0$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = 90°, 270°$

or

$\sin\theta = \sqrt{2}/2$

$\theta = \sin^{-1}(\sqrt{2}/2)$

$= 45°, 135°$

$\therefore$ The value of θ is $45°, 90°, 135°, 270°$

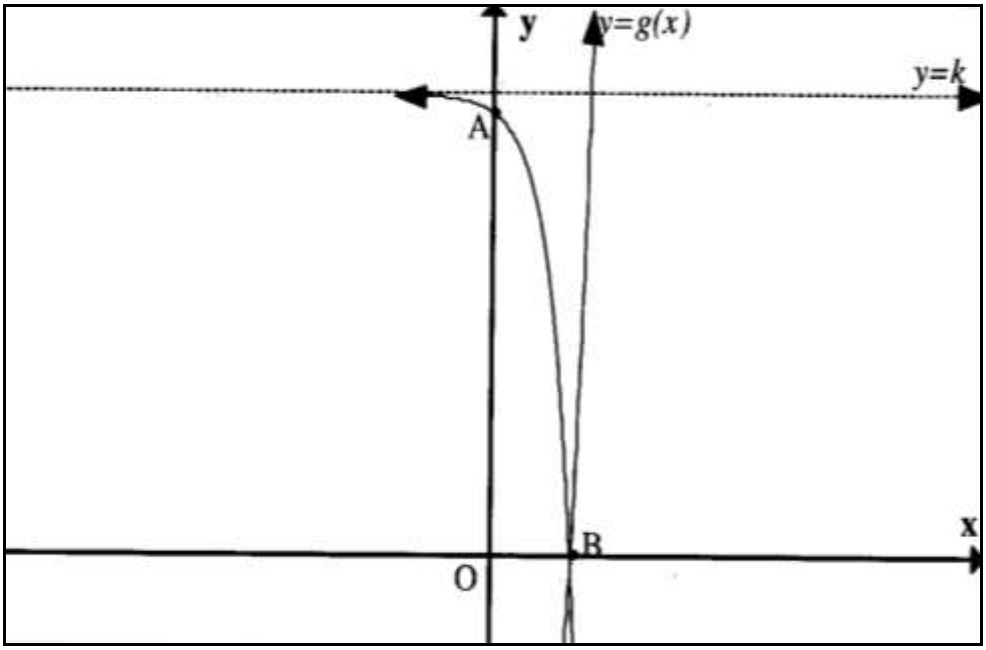
Extract 8.2: A sample of correct responses to part (b) of Question 7

In Extract 8.2, the candidate correctly recalled the identity $\sin 2\theta = 2\sin\theta\cos\theta$.

2.9 Question 9: Exponential and Logarithmic Functions

The question examined the candidate's ability to interpret a graph of exponential functions. The problem read as follows;

The following figure shows part of the curve of the function $g(x) = y$*, where* $g(x) = \left|4e^{2x} - 25\right|$, $x \in \mathbb{R}$.



The curve crosses the y - intercept at point A and meets x - axis at point B.

Find:

(a) *the y coordinate of point A.*

(b) *the x - coordinate of point B.*

(c) *the value of k .*

The candidates' performance was generally weak whereby only 13.18 per cent of the candidates got 3.5 marks or more. Figure 10 shows the percentage of the candidates with low, average and high marks on this question.

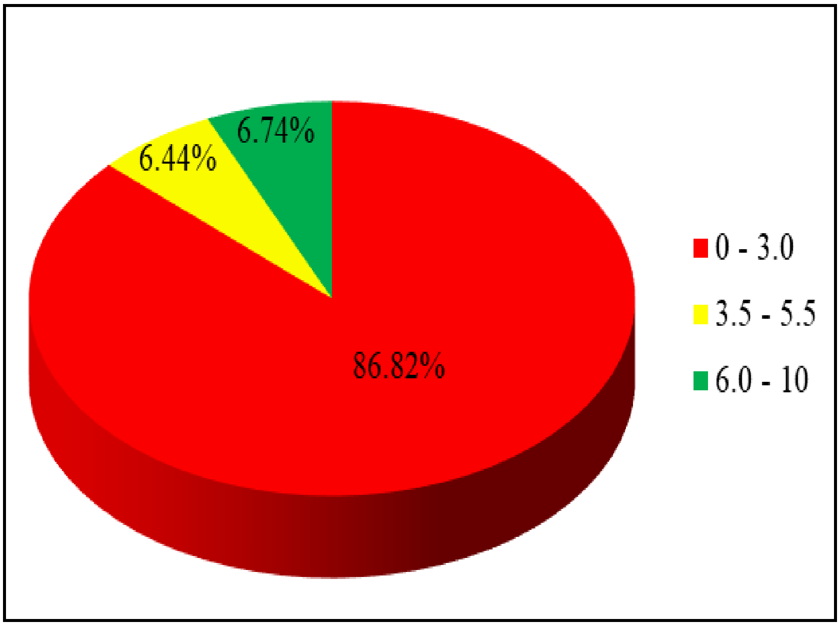


Figure 10: *The overall candidates' performance in Question 9*

As Figure 10 shows, 86.82 per cent of the candidates got low marks. In part (a), most of these candidates misinterpreted the absolute brackets $(|\ |)$. These candidates correctly reached at $g(0)=|-21|$, but they finally got $g(0)=-21$ instead of $g(0)=21$.

In part (b), many candidates failed to solve equations involving natural exponential terms. These candidates were not conversant with the fact that $e^n = A \rightarrow n = \ln A$. The candidates failed to relate natural exponents and logarithms. These candidates wrongly solved $e^x = \frac{5}{4}$ and got $x = \frac{5}{4}$ instead of $x = \ln\left(\frac{5}{2}\right)$.

The responses of the candidates in part (c) indicated that most candidates were not familiar with the properties of exponential functions. They did not realize that $e^{2x} \approx 0$ as $x \rightarrow -\infty$. For example, some of these candidates misinterpreted that $x \rightarrow -\infty$, $e^{2x} \approx 1$ instead of $x \rightarrow -\infty$, $e^{2x} \approx 0$ and therefore, they incorrectly evaluated $|4e^{2x} - 25|$ and got $k = 21$. Further, some candidates did not realize that the curve approaches horizontal asymptote when the value along horizontal axis (x – axis) is a very large negative number (see Extract 9.1).

9. $y = k$.

$y = g(x)$

$|4e^{2x} - 25| = k$.

$4e^{2x} + 25 = k$.

but $x = 0.9$.

$4e^{2(0.9)} - 25 = k$.

$24.19 + 25 = k$.

$k = 49.2$.

$\therefore$ The value of $k = 49.2$.

Extract 9.1: A sample of incorrect responses to part (c) of Question 8

In Extract 9.1, the candidates calculated the value of k by substituting $x = 0.9$ in $\left|4e^{2x} - 25\right|$ instead of a very large negative number.

Despite the weak performance, 196 (0.51%) candidates correctly responded to this question. In part (a) These candidates understood that the abscissa of point A is 0, thus they correctly computed $g(0)$ and got $g(0) = |-21|$. The candidates finally considered the absolute value ending up to $g(0) = 21$.

In part (b), these candidates correctly identified that the ordinate of point B is 0, thus they formulated the equation $0 = \left|4e^{2x} - 25\right|$ and solved it to get $e^x = \frac{5}{2}$ and consequently $x = \ln\left(\frac{5}{2}\right)$. As Extract 9.2 illustrates, in part (c) the candidates correctly evaluated $\left|4e^{2x} - 25\right|$ and got $k = 25$.

(c) Value of k

k is a Horizontal asymptote

- it's a value of y for which $g(x)$ is

Undefined (∞)..

$g(x) = |4e^{2x} - 25|$.

it follows that

$y = 4e^{2x} - 25$

$y \pm 25 = \pm 4e^{2x}$

9(c) Apply ln

$\ln(y\pm25)=(\ln e)x+\ln 8x.$

$\ln(y\pm25)=8x$

$8x=\ln(y\pm25)$

$x=\dfrac{\ln(y\pm25)}{8}$

for x to be Undefined (∞),

$y=25.$

$\therefore\ y=k=25.$

Extract 9.2: A sample of correct responses to part (c) of Question 8

In Extract 9.2, the candidate correctly realized that $e^{2x}\approx 0$ as $x\to-\infty$, thus they correctly evaluated the value of *k*.

2.10 Question 10: Matrices and Linear Programming

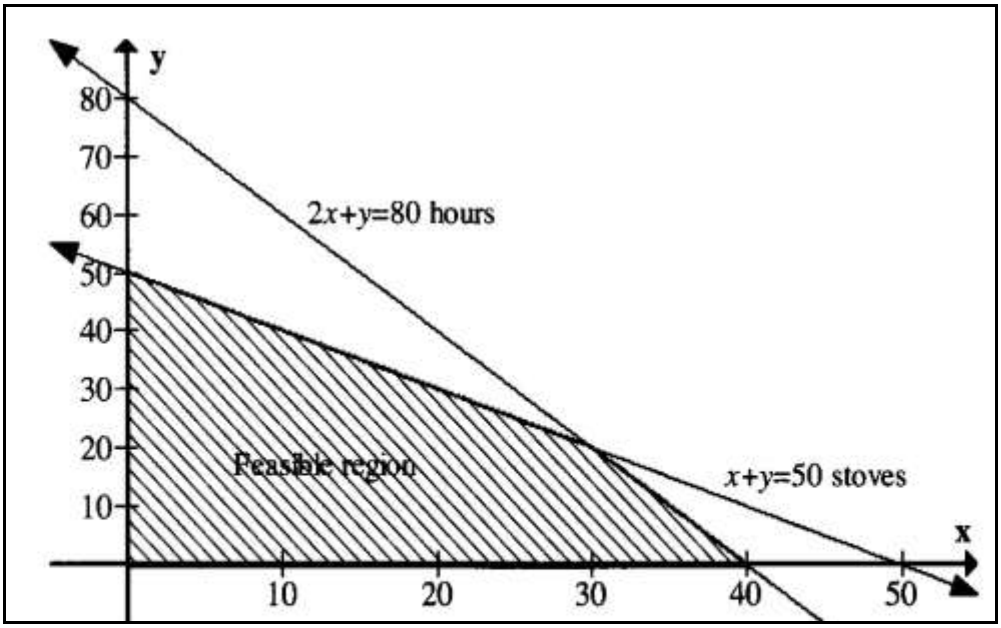
Part (a) of the question was set from the matrix and it assessed the competence of candidates in identifying the size of a matrix and naming the elements in a matrix using row and column. Its sub-parts were:

(i) *Write down all possible orders for a matrix with 6 elements.*

(ii) *Suppose $A=\left[a_{ij}\right]$ is a 2×2 matrix whose elements are given by $a_{ij}=\dfrac{j-i}{2}$. Determine the elements of matrix A.*

Part (b) of this question measured the competence of the candidates to interpret the graph describing a linear programming problem and determine the maximum value of the linear programming problems. The problem read as follows:

The following graph represents business optimization possibilities for a company which sells two types of stove S_1 and S_2. The variable x represents the number of S_1 type while y represent the number of S_2. The time available for the company to make both S_1 type and S_2 type is 80 hours and the space available can hold not more than 50 stoves.



Use the graph to answer the following questions:

(i) *How many hours are used to make one stove of each type?*

(ii) *If one stove of S_1 type is sold at a price of Tshs.300 and one stove of S_2 type is sold at a price of Tshs.200, how many stoves of each type could be sold in order to maximize revenue?*

The performance of the candidates was average since 15,139 (39.47%) candidates scored 3.5 marks or more.

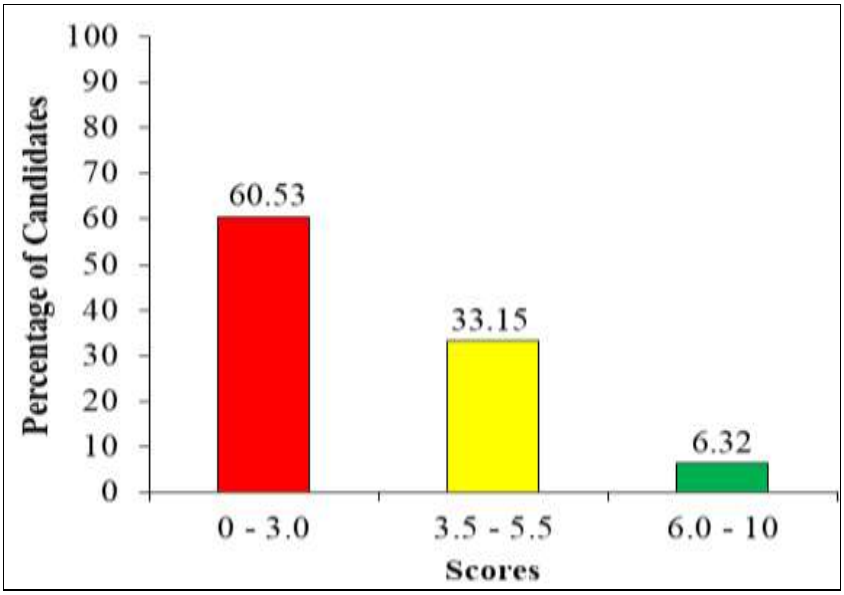


Figure 11: *The candidates' performance in Question 10*

It is further noted that only 55 (0.14%) candidates got full marks. In part (a) (i), the candidates correctly listed the orders; 1×6, 6×1, 3×2 and 2×3. In part (a) (ii) the candidates were conversant that in $A=\lfloor a_{ij}\rfloor$, *i* stands for *i*th row and *j* for *j*th column. They firstly named the elements of 2×2 the matrix $\lfloor a_{ij}\rfloor=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}$ and then correctly substituted the values of *i* and *j* into $a_{ij}=\dfrac{j-i}{2}$ resulting in $a_{11}=0$, $a_{12}=\dfrac{1}{2}$, $a_{21}=-\dfrac{1}{2}$ and $a_{22}=0$.

In part (b) (i), these candidates studied the graph and identified that time is constrained by $2x+y\le 80$. They also realized that the time (in hours) used to make one stove of S_1 type and S_2 type is represented by coefficients of *x* and *y* respectively. Therefore, they correctly concluded that 2 hours are used to make one stove of S_1 type and 1 hour is used to make one stove of S_2 type. In part (b) (ii) these candidates correctly formulated the objective function, maximize: $f(x, y)=300x+200y$. Then, they correctly identified that the corner points of the feasible region are $(0,0)$, $(40,0)$, $(30,20)$ and $(0,50)$. Thereafter, they substituted the points into the objective function and realized that the point $(30,20)$ gives the maximum value of 13,000. Thus, they concluded that the company should sell 30 stoves of S_1 type and 20 stoves of S_2 type to maximize the revenue. Extract 10.1 is a sample of correct responses presented by one of the candidates.

ii) From the graph,
Corner points are $(0, 0)$, $(40, 0)$, $(0, 50)$ and $(30, 20)$
Then, Objective function
$f(x, y) = 300x + 200y$
Evaluation table,

Point	$= 300x + 200y$	Value.
$(0, 0)$	$300(0) + 200(0)$	0.
$(40, 0)$	$300(40) + 200(0)$	12,000/=
$(0, 50)$	$300(0) + 200(50)$	10,000/=
$(30, 20)$	$300(30) + 200(20)$	13,000/=

Therefore, for maximum revenue, 30 S_1 stoves and 20 S_2 stoves should be sold.

Extract 10.1: A sample of the correct responses to part (b) of Question 10.

In Extract 10.1, the candidates correctly identified the corner points, objective function and the number of stoves.

The majority (60.53%) of the candidates scored low marks. In part (a) (i), the candidates wrote various matrices with six elements instead of writing the order itself. For instance, some candidates wrote $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ instead of 2×3 while others wrote $\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$ instead of 3×2. In part (a) (ii), most of these candidates were not familiar with the notation a_{ij} in matrices since many of these candidates skipped this item. Furthermore, some of these candidates applied the definition $a_{ij} = \frac{j-i}{2}$, however, they interchanged the position of i and j resulting in $a_{12} = \frac{1-2}{2}$ instead of $a_{12} = \frac{2-1}{2}$. In part (b) (i), some of these candidates wrongly assumed that the time resource is constrained by $x + y = 50$. Therefore, they incorrectly concluded that 2 hours are used to make one stove of S_1 type. In part (b) (ii), a considerable number of candidates wrote incorrect points such as $(50, 0)$ and $(0, 80)$ that led to an incorrect answer. Furthermore, some of these candidates correctly obtained the optimum solution. However, they concluded by giving the maximum profit as Tshs 13,000 instead of 30 stoves of S_1 type and 20 stoves of S_2 type. Extract 10. 2 represents a sample of incorrect responses to this question.

ii/

Points	$f(x+y)_{max}$ $300x + 200y$
(30, 20)	300(30) + 200(20) = 13000
(0, 80)	300(0) + 200(80) = 16000
(40, 0)	300(40) + 200(0) = 12000
0, 50	300(0) + 200(50) = 10000
50, 0	300(50) + 200(0) = 15000

$\therefore$ 80 stoves of each type should be sold so as to maximize the revenue.

Extract 10.2: A sample of incorrect responses to part (b) of Question 10

In Extract 10.2, the candidates included an incorrect point $(0, 80)$ and consequently obtained incorrect number of stoves (80).

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The tested topics in ACSEE 2023 were *Calculating Devices*, *Functions*, *Algebra*, *Differentiation*, *Integration*, *Statistics*, *Probability*, *Trigonometry*, *Exponential and Logarithmic Functions*, *Matrices* and *Linear Programming*.

The analysis indicates that the candidates' performance was good in 3 topics. These topics are *Statistics* (94.12%), *Calculating Devices* (80.92%) and *Probability* (74.28%). The candidates' good performance in these topics was attributed to the candidates' ability to draw a histogram, use a non-programmable scientific calculator to evaluate mathematical expressions and determine probability of combined events. It is also noted that candidates attained average performance in 4 topics, namely: *Algebra* (55.11%), *Functions* (39.83%), *Matrices* and *Linear Programming* (39.47%).

The overall performance of the candidates in the other 4 topics was weak. These topics include *Exponential and Logarithmic Functions* (13.18%), *Trigonometry* (21.10%), *Integration* (24.61%) and *Differentiation* (29.91%). The weak performance in these topics was due to the failure of the candidates to demonstrate competence in interpreting the graph of the exponential function, recalling compound angle formulae, recalling the properties of definite integrals, recalling the substitution technique to evaluate integrals, determining derivatives of functions and applying differentiation to solve real-life problems. Appendix I shows the candidates' performance in each topic.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

About 68.53 per cent of the candidates passed the examination in 2023, while 58.66 per cent of the candidates passed the examination in 2022. Therefore, there was a performance increase of 9.87 per cent in 2023. In 2023, the candidates' overall performance was good in *Statistics*, *Calculating Devices* and *Probability* topics, while the overall performance in *Algebra*, *Functions*, *Matrices*/*Linear Programming* was average. It is further noted that the performance of the candidates was generally weak in *Exponential and Logarithmic Functions*, *Trigonometry*, *Integration* and *Differentiation*. The comparison of the candidates' performance in each topic for two consecutive years is shown in appendix II.

4.2 Recommendations

In order to improve candidates' performance in the future examination, the following are recommended:

(a) Teaching and learning process should involve real-life activities to enhance learners' ability to solve problems facing the society.

(b) The teaching and learning process should ensure that the learners are also competent in:

(i) drawing and interpreting the graphs of rational, exponential and logarithmic functions and the graphs describing the linear programming problems;

(ii) applying the compound angle formulae to solve trigonometric problems;

(iii) evaluating the definite integrals and using the substitution technique to integrate polynomial functions;

(iv) applying mathematical skills to solve algebraic equations arising in different fields; and

(v) using differentiation to solve real-life problems.

Appendix I

Summary of Candidates' Performance in Each Topic in 141 Basic Applied Mathematics ACSEE 2023

S/N	Topic	Question Number	Percentage of Candidates who Scored 3.5 Marks or More	Remarks
1	Statistics	6	94.12	Good
2	Calculating Devices	1	80.92	Good
3	Probability	7	74.28	Good
4	Algebra	3	55.11	Average
5	Functions	2	39.83	Average
6	Matrices/Linear Programming	10	39.47	Average
7	Differentiation	4	29.91	Weak
8	Integration	5	24.61	Weak
9	Trigonometry	8	21.10	Weak
10	Exponential and Logarithmic Functions	9	13.18	Weak

Appendix II

Candidates' Performance in Each Topic for the year 2022 &2023